Designing for the Road User

Maximum Spiral Transition Lengths

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Abstract

This paper demonstrates that strict adherence to the Transit New Zealand draft State Highway Geometric Design Manual geometric standards can lead to excessively long spiral transition curves that can convey misleading information to the driver. This paper presents a discussion on the influences that the rate of superelevation development and adopted lateral coefficients of friction can have on determining the length of a spiral transition from a tangent to a circular arc. A maximum length of spiral transition is suggested and the necessary corresponding adjustments to the rate of superelevation development are given.

The focus of this paper is on rural two-lane highways with winding alignments and near minimum curve radii measured in relation to the speed environment and design speed.
1 Introduction

The author has noted that a cookbook approach to the geometric design of highways is often adopted. The recommendations and standards contained in manuals are applied steadfastly and unquestioningly without fully appreciating the background and reasoning behind them. Manuals seldom differentiate clearly what the relative importance of each recommendation or standard is. In most practical geometric designs all the recommendations and standards cannot be met simultaneously. Inevitably some of them have to be ignored or discarded to arrive at a practical design solution. The inexperienced designer, faced with an over-prescription of such requirements, tends sometimes to adopt a recommendation of relatively minor importance at the expense of a fundamental principle, usually resulting in a compromised or unsafe design.

This paper demonstrates that strict adherence to the Transit New Zealand draft State Highway Geometric Design Manual (SHGDM) geometric standards can lead to excessively long spiral transition curves that can convey misleading information to the driver. This paper presents a discussion on the influences that the rate of superelevation development and adopted lateral coefficients of friction can have on determining the length of a spiral transition from a tangent to a circular arc. A maximum length of spiral transition is suggested and the necessary corresponding adjustments to the rate of superelevation development are given.

The focus of this paper is on rural two-lane highways with winding alignments and near minimum curve radii measured in relation to the speed environment and design speed.

It is assumed that the reader understands the basic principles of horizontal highway design geometry.

2 Spiral transition lengths

The SHGDM standard rate for the development of superelevation (2.5 % / s of travel time) can lead to long spiral transitions that hide the true radius of a curve from the driver’s view if not applied judiciously. Hidden curves have safety implications. They result in surprise, last-minute adjustments to speed, and eventually cause driver fatigue and error if there are too many decisions to be made. 1

With long transitions the circular arc portion of curve is hidden from the driver’s view at the point on the tangent where the driver must assess the radius of the circular arc and start to adjust the speed and path of the vehicle accordingly. With excessively long transitions, the portion of the transition curve that is visible from the decision point on the tangent can represent a significantly larger radius than the radius of the circular arc 2.

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1 The author has counted 300 curves in 100 km on SH8 between Lawrence and Alexandra. Approximately 200 were not fully visible from the tangent.
2 The example in Figure 2-1 is a true to scale drawing of a 230 m circular arc with a spiral transition length of 100 m, based on the SHGDM standards for a 90 km/h design speed with maximum superelevation of 10 %. The radius of the spiral at the focal point is 300 m.
In these cases, drivers have to reassess the situation once they are in the curve and the true radius becomes visible. Apart from being frustrating and tiring on winding roads, especially on roads that do not have a consistent design speed, a potentially dangerous situation arises when a driver cannot decelerate sufficiently under engine compression and has to brake in the curve.

Long spiral transition lengths also cause difficulty in designing curves with sufficiently long circular arcs in relation to the spiral transitions, especially for smaller deflection angles. Short arc lengths and long transition lengths encourage cutting across the corner into the face of oncoming traffic. Therefore a limit is often placed on the ratio of circular arc length to transition length.

### 2.1 Maximum spiral transition length

The SHGDM is silent on the maximum transition length, as are many other manuals, but some do allude indirectly to the problem of excessive spiral transition lengths. The California Department of Transportation Highway Design Manual (Caltrans HDM) states simply that ‘Spiral transition curves are not standard practice.’ [California Department of Transportation, 2006, p. 200-18] The United Kingdom Highways Agency Design Manual for Roads and Bridges (DMRB) states that ‘...the length of transition should normally be limited to $\sqrt{24R}$ metres.’ [The Highways Agency, 2002, p. 3/3]

The author suggests from observation 3 that the focal point for assessing the curve radius from the decision point on the tangent lies within the lane width and on the projection of the tangent. Refer to Figure 2-1. Beyond this point the foreshortening effect of the low viewing angle and the reduction in size of image caused by the diminishing perspective makes it increasingly difficult to judge the radius accurately, until eventually the curve disappears from the view. It is also important that a driver can see a reasonable length of the curve to assess its radius, not just a point.

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3 During the formulation of this observation, the author drove approximately 60,000 km on New Zealand’s rural State highways SH6, SH8, and SH85 in two years,
The author suggests that the offset of the start of the circular arc (SC) from the projected tangent should be approximately equal to the lane width (3.5 m to 3.7 m). The road edge markings provide the most visual impact, but the shoulder edge also provides some delineation and some of the shoulder width can be included in the view (0.5 m). Therefore, a 4 m maximum offset of the SC point from the projected tangent would provide sufficient visible length of the transition curve that is close enough to the circular arc radius for drivers to make a correct assessment of its radius while they are still on the tangent.

The offset \((YC)\) to the SC point is given by:

\[
YC \approx SL \left[ \frac{\phi_{spiral}}{3} \right]
\]

where: \(SL = \) spiral transition length (m),

and \(\phi_{spiral} = \) spiral angle (radians).

The spiral angle is given by:

\[
\phi_{spiral} = \frac{SL}{2R}
\]

where: \(R = \) circular curve radius (m).

Therefore:

\[
YC \approx \frac{SL^2}{6R}
\]

But the lateral shift \((P)\) of the circular arc away from the tangent is:

\[
P = \frac{SL^2}{24R}
\]

Therefore:

\[
P \approx \frac{YC}{4}
\]

For a maximum offset of 4 m the lateral shift \((P)\) should therefore be a maximum of 1 m.

Therefore for \(P = 1.0\) m:

**Equation 2-1** \(\quad SL_{max} = \sqrt{24R}\)

Coincidentally, Equation 2-1 is the same as the recommended maximum length of transition suggested in the DMRB. [The Highways Agency, 2002, p.3/3]
Equation 2-1 produces the upper curve indicated in Figure 2-2, above which transition lengths would cause the SC to be offset out of view from an observation point on the tangent.

![Figure 2-2: SHGDM spiral transition lengths](image)

### 2.2 Minimum spiral transition length

Most manuals concur that transition lengths that result in horizontal shifts less than 0.25 m for a given arc radius are not necessary. They provide no noticeable difference in the transition from tangent to circular arc. [Transit New Zealand, 2005, p.4-8] Therefore for $P = 0.25$ m:

**Equation 2-2**  
\[ SL_{\text{min}} = \sqrt{6R} \]

Equation 2-2 produces the lower curve indicated in Figure 2-2, below which the transition lengths would be too small to provide a noticeable transition.

### 2.3 Determining spiral transition length

The SHGDM prescribes the following method for determining the length of spiral transition curves. [Transit New Zealand, 2005, p.2-21, 4-8, & 4-9]

The crown runoff from 3 % normal camber to 0 % is positioned on the tangent immediately preceding the start of the spiral transition (TS). The development length from 0 % to full
superelevation coincides exactly with the length of the spiral transition. At the start of the circular arc (SC) the superelevation is fully developed.

The minimum crown runoff length and minimum transition length are calculated using the design speed and a maximum rotation rate of 2.5 % / s of travel time, which is based primarily on aesthetic appearance and the comfort of the vehicle occupants. The SHGDM allows the use of 3.5 % / s in constrained conditions, but only for design speeds less than 80 km/h unless specific approval is granted. The Austroads Guide to the Geometric Design of Rural Roads recommends the same development rates under essentially the same conditions. [Austroads, 2003, p.43]

The resulting SHGDM relationships between radius, length of transition, and superelevation for a constant rate of superelevation development are shown in Figure 2-2. The derivations of the formulae that were used to plot the curves in Figure 2-2 are given in Appendix A.

The SHGDM approach, while being theoretically sound and logical, returns long spiral transitions in relation to the circular arc radii. This is shown towards the upper left-hand side of Figure 2-2 where the set of curves extends above the cut-off line representing Equation 2-1, this being the limit beyond which the true curve radius will not be visible. The combinations of design speeds and radii represented in the area above the cut-off line are common on many of New Zealand’s more remote rural highways due to the winding nature of the roads, the hilly topography of New Zealand, and a need to minimise the cost of construction.

It is necessary to consider why the SHGDM returns such long spiral transition lengths, when this does not appear to be a concern in the geometric design manuals of other countries. The answer is two-fold; and is related to the adopted values of maximum allowable lateral side friction coefficients and the rate of superelevation development. These are discussed in the following two sections.

2.4 Lateral coefficient of friction

The formula for the minimum radius for a given design speed is:

\[
R_{\text{min}} = \frac{V_{\text{design}}^2}{127(e_{\text{max}} + f_{\text{max}})}
\]

The greater the maximum superelevation \(e_{\text{max}}\) and the greater the maximum allowable lateral friction coefficients \(f_{\text{max}}\) that are adopted, the smaller the minimum radius will be for any given design speed \(V_{\text{design}}\).

The SHGDM has adopted less conservative lateral coefficients of friction than manuals of other countries for determining the minimum radius of curves in the lower range of design speeds. In Table 2-1 the SHGDM standards [Transit New Zealand, 2005, p.2-24] are compared with those of the American Association of State Highway and Transportation Officials (AASHTO) A Policy on Geometric Design for Highways and Streets 4th Edition (Green Book) standards [AASHTO, 2001, Exhibit 3-14, p.145] and also on research by the National Cooperative Highways Research Programme (NCHRP) in its Report 439. [New
The maximum superelevation in all cases is 10%.

<table>
<thead>
<tr>
<th>$V_{\text{design}}$ km/h</th>
<th>SHGDM</th>
<th>AASHTO Green Book</th>
<th>NCHRP 439</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{\text{max}}$ m/m</td>
<td>$f_{\text{max}}$ m/m</td>
<td>$R_{\text{min}}$ m</td>
</tr>
<tr>
<td>60</td>
<td>0.10</td>
<td>0.33</td>
<td>66</td>
</tr>
<tr>
<td>80</td>
<td>0.10</td>
<td>0.26</td>
<td>140</td>
</tr>
<tr>
<td>100</td>
<td>0.10</td>
<td>0.14</td>
<td>328</td>
</tr>
<tr>
<td>120</td>
<td>0.10</td>
<td>0.11</td>
<td>540</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$V_{\text{design}}$ km/h</th>
<th>SHGDM</th>
<th>DMRB Absolute Minimum</th>
<th>DMRB Desirable Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{\text{max}}$ m/m</td>
<td>$f_{\text{max}}$ m/m</td>
<td>$R_{\text{min}}$ m</td>
</tr>
<tr>
<td>60</td>
<td>0.10</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>80</td>
<td>0.10</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>100</td>
<td>0.10</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>120</td>
<td>0.10</td>
<td>0.07</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 2-1: Lateral demand coefficients of friction and minimum radii

In Table 2-2 the United Kingdom Highways Agency Design Manual for Roads and Bridges (DMRB) standards [The Highways Agency, 2002, p.3/1] are more conservative than SHGDM [Transit New Zealand, 2005, p2-24] in the choices of both maximum superelevation ($e_{\text{max}} = 5\%$ to $7\%$) and demand coefficients of friction.

The SHGDM coefficients of friction are the same as the Austroads Guide to the Geometric Design of Rural Roads absolute maximum coefficients except for a difference of -0.02 at 90 km/h and 100 km/h. [Austroads, 2003, p.37] The Austroads Guide to the Geometric Design of Rural Roads desirable maximum coefficients of lateral friction tend towards the values adopted by AASHTO Green Book and those proposed in the NCHRP 439 report, but are not as conservative as either AASHTO Green Book or NCHRP 439.

The coefficients of friction adopted in SHGDM result in smaller minimum design radius curves, especially for design speeds less than 100 km/h. As illustrated in Table 2-1 and in Table 2-2 the resulting SHGDM minimum design radii can be as little as half the radii produced by the values adopted in the manuals of other countries.

The less conservative approach of the SHGDM need not necessarily be regarded as a deficiency. New Zealand is a developing country with challenging topography, and it needs...
to provide roads economically. New Zealand has sensibly adopted similar coefficients of friction as other countries at design speeds greater than 100 km/h, which would typically be applicable to its major highways, but has realised the advantages of a less conservative approach for lower design speeds. Nevertheless, the consequence is that New Zealand curves are sensitive, as far as visibility is concerned, to the length of the spiral transition.

2.5 Superelevation Development Rate

Superelevation development rate can be expressed as either the rate of rotation of the road surface or as the relative gradient of the edge of the road to the centre line. The rate of rotation can be viewed as a representation of the comfort parameter, and the relative gradient as a representation of the aesthetic parameter. Most design manuals seem to adopt the parameter to which their users have grown accustomed ⁴.

In contrast to the less conservative approach in adopting a range of coefficients of friction for various design speeds, the SHGDM has adopted a conservative uniform superelevation development rate of 2.5 % / s of travel time for two-lane roads across all design speeds. The SHGDM does allow the requirement to be relaxed to 3.5 % / s in constrained conditions, but only for design speeds less than 80 km/h. [Transit New Zealand, 2005, p.2-21]

In Table 2-3 and Table 2-4, the SHGDM lengths of spiral transition, development rate, and relative gradient are compared with those produced using the manuals of other countries.

The AASHTO Green Book recommends relative gradients of 1:155 at 50 km/h to 1:285 at 130 km/h [AASHTO, 2001, Exhibit 3-24, p.162]. The Caltrans HDM recommends a uniform relative gradient of 1:150 for all multi-lane carriageways, and 1:150 based on a 5 m width for two-lane roads. [California Department of Transport, 2006, p.200-13] This is equivalent to 1:215 for a 3.5 m width. In Table 2-3 the relative gradients have been converted to an equivalent rate of rotation based on a lane width of 3.5 m for comparison with the SHGDM standard. In each case the maximum superelevation is 10 %.

<table>
<thead>
<tr>
<th>$V_{design}$ km/h</th>
<th>SHGDM</th>
<th>AASHTO Green Book</th>
<th>Caltrans HDM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SL</td>
<td>Dev rate</td>
<td>Relative gradient</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>% / s</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>67</td>
<td>2.5</td>
<td>1:190</td>
</tr>
<tr>
<td>80</td>
<td>89</td>
<td>2.5</td>
<td>1:250</td>
</tr>
<tr>
<td>100</td>
<td>111</td>
<td>2.5</td>
<td>1:320</td>
</tr>
<tr>
<td>120</td>
<td>133</td>
<td>2.5</td>
<td>1:380</td>
</tr>
</tbody>
</table>

Table 2-3: Comparison of superelevation development rates

⁴ There are exceptions, such as the South African NRA geometric design guidelines, which does differentiate between rate of rotation and relative gradient. [South African National Road Agency Ltd, 2003, p.4-17]

⁵ Because Caltrans HDM does not recommend using spiral transitions, the lengths given in Table 2-3 are for the superelevation runoff. Two thirds of the length falls on the tangent and the remainder in the circular arc. For two-lane roads $L = 750 \times e$.

⁶ The relative gradient for two-lane roads with a lane width of 3.5 m and $e = 10 \%$ is 75 m divided by 0.35 m.
The DMRB bases its development length on an acceptable rate of change of centripetal acceleration of \( q = 0.3 \text{ m/s}^3 \) generally, increasing to \( q = 0.6 \text{ m/s}^3 \) in ‘difficult cases.’ [The Highways Agency, 1993, p.3/3] The basic transition length is derived from the following formula.

\[
SL = \frac{V_{\text{design}}^3}{46.7 \times q \times R}
\]

Converting the rate of change of centripetal acceleration into rate of development gives 1.4 % / s to 2.8 % / s. The rates do not vary across the range of design speeds. There is an overriding recommendation in the DMRB that the length of transition should be limited to a shift of \( P = 1.0 \text{ m} \). This means that for \( e_{\text{max}} = 7 \% \), the rate of change of centripetal acceleration must be increased to \( q = 0.6 \text{ m/s}^3 \) to limit the transition lengths as indicated in Table 2-4.

<table>
<thead>
<tr>
<th>( V_{\text{design}} ) km/h</th>
<th>SHGDM ( (e_{\text{max}} = 10 %) )</th>
<th>DMRB Desirable Minimum ( (e_{\text{max}} = 5 %) ) (( q = 0.3 \text{ m/s}^3 ))</th>
<th>DMRB Absolute Minimum ( (e_{\text{max}} = 7 %) ) (( q = 0.6 \text{ m/s}^3 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( SL ) m</td>
<td>Dev rate % / s</td>
<td>Relative gradient</td>
</tr>
<tr>
<td>60</td>
<td>67</td>
<td>2.5</td>
<td>1:190</td>
</tr>
<tr>
<td>80</td>
<td>89</td>
<td>2.5</td>
<td>1:250</td>
</tr>
<tr>
<td>100</td>
<td>111</td>
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<td>1:320</td>
</tr>
<tr>
<td>120</td>
<td>133</td>
<td>2.5</td>
<td>1:380</td>
</tr>
</tbody>
</table>

Table 2-4: Comparison of superelevation development rates

### 3 Revision of superelevation development rate

If in New Zealand lower minimum radii need to be applied for economic reasons, and if the superelevation development length should be matched to the transition length, then a compromise has to be made. Either many of the SC points must be hidden from the driver’s view at near minimum standards, or the maximum rate of superelevation development must be relaxed so that shorter spiral transition lengths can be applied to designs. It is not in the interests of safety to accept that the radius of a curve should be obscured. It therefore remains that the appropriateness of the comfort and aesthetic based maximum rate of superelevation development should be reassessed.

From Equation 2-1 a shift of \( P = 1.0 \text{ m} \) limits the transition length to:

\[
SL_{\text{max}} = \sqrt{24R}
\]
The relationship between the rate of rotation and transition length is given by:

\[ SL = \frac{V_{design} \times e}{3.6 \times \text{development rate}} \]

Combining the two equations and rearranging for the development rate that would limit the shift to 1 m gives:

**Equation 3-1**

\[ \text{Development rate} = \frac{V_{design} \times e_{max}}{3.6 \times \sqrt{24R_{min}}} \]

Table 3-1 contains adjusted development rates calculated from Equation 3-1 that will always limit the shift \((P)\) to 1.0 m at minimum radii. The equivalent relative gradients based on a lane width of 3.5 m are included in Table 3-1 for comparison with other standards.

<table>
<thead>
<tr>
<th>(V_{design})</th>
<th>(R_{min})</th>
<th>Development rate</th>
<th>Relative gradient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{km/h})</td>
<td>(\text{m})</td>
<td>(\text{m})</td>
<td>(% / \text{s})</td>
</tr>
<tr>
<td>50</td>
<td>44</td>
<td>32</td>
<td>4.3</td>
</tr>
<tr>
<td>60</td>
<td>66</td>
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<td>4.2</td>
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<td>70</td>
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<td>48</td>
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<td>80</td>
<td>140</td>
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<td>3.8</td>
</tr>
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<td>90</td>
<td>228</td>
<td>74</td>
<td>3.4</td>
</tr>
<tr>
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<td>89</td>
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<tr>
<td>110</td>
<td>433</td>
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<td>3.0</td>
</tr>
<tr>
<td>120</td>
<td>540</td>
<td>114</td>
<td>2.9</td>
</tr>
</tbody>
</table>

\(e_{max} = 10\%\)

**Table 3-1 : Adjusted development rates**

4 Discussion

The adjusted development rates in Table 3-1 (4.3 % / s to 2.9 % / s) are marginally greater than the SHGDM range from 2.5 % / s in normal situations to 3.5 % / s in constrained conditions.

The range of development rates in Table 3-1 (4.3 % / s to 2.9 % / s) compares favourably with the ranges recommended by the AASHTO Green Book (2.5 % / s to 3.6 % / s) and Caltrans HDM (1.9 % / s to 4.4 % / s).

The adjusted development rates for design speeds greater than 100 km/h (3.0 % / s to 2.9 % / s) compare favourably with the rate of 2.8 % / s used in the DMRB where \(q = 0.6 \text{ m/s}^3\).
The adjusted development rates in Table 3-1 range from a higher rate of 4.3 % / s at 50 km/h to a lower rate of 2.9 % / s at 130 km/h. This is in keeping with the assumption that vehicle occupants should expect, and would probably accept, a greater level of discomfort at lower design speeds than higher design speeds. Compare this with the values used by Caltrans HDM in Table 2-3, which are the reverse.

The same argument applies to the aesthetic component. The relative gradients in Table 3-1 are steep for low design speeds (1:90 at 50 km/h) and flat for high design speeds (1:350 at 130 km/h), which is what would be expected to be in keeping with the standard of the road.

The range of relative gradients in Table 3-1 compares favourably with the relative gradients adopted by the manuals of other countries (DMRB: 1:100 to 1:370, AASHTO Green Book: 1:155 to 1:285, Caltrans HDM: 1:215, and Austroads Guide to the Geometric Design of Rural Roads, 1:111 to 1:250 [Austroads, 2003, p.44]).

Figure 4-1 has been produced using the same values for the variables as in Figure 2-2, with the exception that the adjusted development rates from Table 3-1 have been substituted instead of using the SHGDM development rate of 2.5 % / s. All the curves in Figure 4-1 now lie below the upper limit of transition lengths.

**Figure 4-1 : Adjusted spiral transition lengths**
Note that both Figure 2-2 and Figure 4-1 return the exactly the same design speed / radius / superelevation interrelated values. The only difference between the curves in Figure 2-2 and Figure 4-1 is the variable rate of development that was applied for each design speed to calculate the transition lengths.

It is not necessary to replace Figure 2-2 with Figure 4-1. The SHGDM can continue to be used based on a development rate of 2.5 % / s, but if the calculated transition length falls above the $P = 1.0$ m limit, then the length should be adjusted using the rates of rotation given in Table 3-1. This will ensure that comfort, aesthetics, and safety can be accommodated in all cases.

5 Conclusions

The fundamental approach to determining transition lengths in SHGDM is acceptable. However, the maximum development rate of 2.5 % / s is unsatisfactory at near minimum radii and near maximum superelevation. The rate returns transition lengths that are long in relation to the radii of the circular arcs, which can obscure the true radius of a curve from the driver positioned on the tangent.

The adjusted development rates, which are based on limiting the shift of the spiral transition to 1.0 m, compare favourably with the development rates and relative gradients used in the USA and United Kingdom.

6 Recommendation

It is recommended that a range of higher superelevation development rates be adopted for different design speeds so that the shift of the transition spiral is limited to 1.0 m. This will limit the offset of the SC to a maximum of 4 m from the projected tangent, and will ensure that the circular arc is always visible from a decision point on the tangent.

This approach needs to be followed only when the transition shift is greater than 1.0 m. In all other cases the 2.5 % / s rate of rotation can be used without any problems.
References


Publications are referenced in the text using the format [author, year of publication, page number].
Appendix A : Derivation of Figure 2-2

Arc Radius and Transition Length

The standards in the draft SHGDM have been reproduced in Figure 2-2. The curves depict the relationship between circular arc radius, superelevation, design speed, and minimum transition length. The lengths are based on a maximum superelevation rotation rate of 2.5 % / s of travel time for the development from 0 % crossfall to full superelevation.

The following sections explain the derivation the curves in Figure 2-2.

Maximum Superelevation Curve

The uppermost superelevation curve in Figure 2-2 (\( e_{\text{max}} = 10 \% \)) is derived from the direct relationship between the minimum radii (SHGDM Table 2.9) for a maximum superelevation rate (SHGDM Table 2.9) at various design speeds. At each design speed the minimum length required for development of superelevation at 2.5 % / s is computed.

\[
SL_{\text{min}} = \frac{e_{\text{max}}}{2.5\%} \times \frac{V_{\text{design}}}{3.6}
\]

Example

\( V_{\text{design}} = 80 \text{ km/h} \)
\( e_{\text{max}} = 10 \% \quad \text{SHGDM Table 2.9} \)
\( R_{\text{min}} = 140 \text{ m} \quad \text{SHGDM Table 2.9} \)

Therefore

\( SL_{\text{min}} = 89 \text{ m} \)

Superelevation Curves

The curves in Figure 2-2 for superelevation values less than the maximum are derived from relationship between superelevation and radius greater than the minimum.

\[
e = \frac{V_{\text{design}}^2 \times k}{1.27R}
\]

where

\[
k = \frac{e_{\text{max}}}{e_{\text{max}} + f_{\text{max}}}
\]

At each design speed the minimum length required for development of superelevation at 2.5 % / s is calculated.
\[ SL_{\text{min}} = \frac{e_{\text{max}}}{2.5\%} \times \frac{V_{\text{design}}}{3.6} \]

Example

\[ V_{\text{design}} = 80 \text{ km/h} \]
\[ R = 200 \text{ m} \]
\[ e_{\text{max}} = 0.10 \]
\[ f_{\text{max}} = 0.26 \quad \text{SHGDM Table 2.9} \]

Therefore
\[ k = 0.278 \quad \text{SHGDM Table 2.8} \]

and
\[ e = 7.0 \% \]

and
\[ SL = 62 \text{ m} \]

**Design Speed Curves**

The same equations that were used in the preceding sections were rearranged with \( V_{\text{design}} \) on the left-hand side of the equation to produce the design speed curves in Figure 2-2.

**Using Figure 2-2**

The intersection of the design speed curve with a chosen radius will return the required superelevation at that radius, and the minimum transition length.

Example

\[ V_{\text{design}} = 100 \text{ km/h} \]
\[ R = 400 \text{ m} \]

Therefore
\[ e = 8.2 \% \quad \text{(interpolate between } e = 8 \% \text{ and } 9 \%) \]

and
\[ SL_{\text{min}} = 91 \text{ m} \quad \text{(read off y-axis)} \]